

according to my thermo-dynamic estimate. This is as good an agreement as could be expected when the difficulties of the investigations, and the uncertainty which still exists as to the true measure of the electromotive force of the Daniell's element are considered. It must indeed be remarked that the electromotive force of Daniell's battery varies by two or three or more per cent. with variations of the solutions used; that it varies also very sensibly with temperature; and that it seems also to be dependent, to some extent, on circumstances not hitherto elucidated. A thorough examination of the electromotive force of Daniell's and other forms of galvanic battery, is an object of high importance, which it is to be hoped will soon be attained. Until this has been done, at least for Daniell's battery, the results of the preceding paper may be regarded as having about as much accuracy as is desirable.

I may state therefore, in conclusion, that the average electromotive force per cell of the Daniell's batteries which I have used, produces a difference of potentials* amounting to $\cdot 0021$ in British electrostatic measure. This statement is perfectly equivalent to the following in more familiar terms:—

One thousand cells of Daniell's battery, with its two poles connected by wires with two parallel plates of metal $\frac{1}{100}$ th of a foot apart and each a square foot in area, produces an electrical attraction equal to the weight of 55 grains.

II. "Measurement of the Electromotive Force required to produce a Spark in Air between parallel metal plates at different distances." By Professor W. THOMSON, F.R.S.
Received January 26, 1860.

The electrometers used in this investigation were the absolute electrometer and the portable electrometer described in my last communication to the Royal Society, and the operations were executed by the same gentlemen, Mr. Smith and Mr. Ferguson. The conductors between which the sparks passed were two unvarnished plates of a condenser, of which one was moved by a micrometer screw, giving a motion of $\frac{1}{25}$ of an inch per turn, and having its

* See §§ 10, 11 of Appendix to the following communication.

head divided into 40 equal parts of circumference. The readings on the screw-head could be readily taken to tenth parts of a division, that is to say, to $\frac{1}{10,000}$ of an inch on the distance to be measured. The point from which the spark would pass in successive trials being somewhat variable and often near the edges of the discs, a thin flat piece of metal, made very slightly convex on its upper surface like an extremely flat watch-glass, was laid on the lower plate. It was then found that the spark always passed between the crown of this convex piece of metal and the flat upper plate. The curvature of the former was so small, that the physical circumstances of its own electrification near its crown, the opposite electrification of the opposed flat surface in the parts near the crown of the convex, and the electric pressure on or tension in the air between them could not, it was supposed, differ sensibly from those between two plane conducting surfaces at the same distance and maintained at the same difference of potentials.

The reading of the screw-head corresponding to the position of the moveable disc, was always determined electrically by making a succession of sparks pass, and approaching the moveable disc gradually by the screw until all appearance of sparks ceased. Contact was thus produced without any force of pressure between the two bodies capable of sensibly distorting their supports.

With these arrangements several series of experiments were made, in which the differences of potentials producing sparks across different thicknesses of air were measured first by the absolute electrometer, and afterwards by the portable torsion electrometer. The following Tables exhibit the results hitherto obtained.

TABLE I.—December 13, 1859. Measurements by absolute electrometer of maximum electrostatic forces* across a stratum of air of different thicknesses.

Area of each plate of absolute electrometer= $\cdot 187$ of a square foot.

Distance between plates of absolute electrometer= $\frac{1}{60}$ of a foot.

Length of spark in inches. <i>s.</i>	Weight in grains required to balance in absolute electrometer. <i>w.</i>	Electromotive force in units of the electrometer. $\sqrt{w}$.	Electrostatic force, or electromotive force per inch of air, in temporary units. $\frac{\sqrt{w}}{s}$.
$\cdot 007$	6	2·4495	349·9
$\cdot 0105$	9	3·0000	285·7
$\cdot 0115$	10	3·1622	275·0
$\cdot 014$	13	3·6055	257·5
$\cdot 017$	16	4·0000	235·3
$\cdot 018$	19	4·3589	242·2
$\cdot 024$	30	5·4772	228·2
$\cdot 0295$	40	6·3245	214·4
$\cdot 034$	50	7·0710	208·0
$\cdot 0385$	60	7·7459	201·2
$\cdot 041$	70	8·3666	204·1
$\cdot 0445$	80	8·9442	201·0
$\cdot 048$	90	9·4868	197·6
$\cdot 052$	100	10·0000	192·3
$\cdot 055$	110	10·4880	190·7
$\cdot 058$	120	10·9544	188·9
$\cdot 060$	130	11·4017	190·0

These numbers demonstrate an unexpected and a very remarkable result,—that greater electromotive force per unit length of air is required to produce a spark at short distances than at long. When it is considered that the absolute electrification of each of the opposed surfaces† depends simply on the electromotive force per unit length of the space between them, or, which is the same thing, the resultant electrostatic force in the air occupying that space, it is difficult even to conjecture an explanation. Without attempting to explain it, we are forced to recognize the fact that a thin stratum of air is stronger than a thick one against the same disruptive tension in the air, according to Faraday's view of its condition as transmitting electric force, or against the same lifting electric pressure from its bounding surfaces, according to the views of the 18th cen-

* See § 3 below.

† See § 4 below.

tury school, as represented by Poisson. The same conclusion is established by a series of experiments with the previously-described portable torsion electrometer substituted for the absolute electrometer, leading to results shown in the following Table.

TABLE II.—January 17, 1860. Measurements by portable torsion electrometer of electromotive forces producing sparks across a stratum of air of different thicknesses.

Length of spark in inches. s.	Torsion in degrees required to balance in electrometer. θ .	Electromotive force in units of the electrometer. $\sqrt{\theta}$.	Electrostatic force, or electromotive force per inch of air, in tem- porary units. $\sqrt{\theta} \div s$.
·001	3	1·732	1732
·002	7	2·646	1323
·003	11	3·316	1105
·004	14	3·742	935
·005	18	4·243	849
·006	22	4·690	782
·007	27	5·196	742
·008	30	5·477	685
·009	33	5·744	638
·010	38	6·164	616
·011	43	6·557	596
·012	48·5	6·964	580
·013	54	7·348	565
·014	59	7·681	549
·015	66	8·124	542
·016	73	8·544	534
·017	79	8·888	523
·018	85	9·219	512

The series of experiments here tabulated stops at the distance 18 thousandths of an inch, because it was found that the force in the electrometer corresponding to longer sparks than that, was too strong to be measured with certainty by the portable electrometer, whether from the elasticity of the platinum wire, or from the rigidity of its connexion with the aluminium index being liable to fail when more than 85° or 90° of torsion were applied. So far as it goes, it agrees remarkably well with the other experiments exhibited in Table I., as is shown by the following comparative Table, in which, along with results of actual observation extracted from Table II., are placed results deduced from Table I. by interpolation for the same lengths of spark.

TABLE III. Experiments of December 13, 1859, and January 17, 1860, compared.

Col. 1. Length of spark in inches. s.	Col. 2. Electromotive force per inch of air, . Dec. 13, in tempo- rary units of that day. $\frac{\sqrt{w}}{s}$	Col. 3. Electromotive force per inch of air, Jan. 17, in tem- porary units of that day. $\frac{\sqrt{\theta}}{s}$	Col. 4. Ratios of numbers in Col. 3. to numbers in Col. 2.
·007	349·3	742	2·13
·0105	285·7	606	2·12
·0115	275·0	588	2·14
·014	257·5	549	2·14
·017	235·3	523	2·22
·018	242·2	512	2·11
			Mean 2·14

The close agreement with one another of the numbers in Col. 4, derived from series differing so much as those in Cols. 2 and 3, and obtained by means of electrometers differing so much in construction, constitutes a very thorough confirmation of the remarkable result inferred above from the experiments of the first series, and shows that the law of variation of the electrostatic force in the air required to produce sparks of the different lengths, must be represented with some degree of accuracy by the numbers shown in the last column of either Table I. or Table III.

The following additional series of experiments were made on precisely the same plan as those of Table II.

TABLE IV.—January 21, 1860. Measurements by portable torsion electrometer of electromotive forces producing sparks across a stratum of air of different thicknesses.

Length of spark in inches. s .	Torsion in degrees required to balance in electrometer. θ .	Electromotive force in units of the electrometer. $\sqrt{\theta}$.	Electrostatic force, or electromotive force per inch of air, in tem- porary units. $\sqrt{\theta} \div s$.
·001	3·2	1·79	1790
·002	6·4	2·32	1160
·003	10·5	3·24	1080
·004	13·2	3·63	907
·005	14·2	3·77	754
·006	18·2	4·27	712
·007	21·7	4·66	666
·012	41·2	6·42	535
·013	46·7	6·83	525
·014	53·2	7·29	521
·015	57·2	7·56	504
·016	63·2	7·95	497
·017	68·2	8·26	486
·018	78·2	8·84	491

TABLE V.—January 23, 1860. Similar experiments repeated.

s .	θ .	$\sqrt{\theta}$.	$\sqrt{\theta} \div s$.
·001	3·5	1·87	1870
·002	6·5	2·55	1275
·003	9·5	3·08	1027
·004	12·7	3·56	890
·005	15·5	3·94	788
·006	18·5	4·30	716
·007	23·0	4·80	686
·008	25·62	5·06	632
·009	30·5	5·52	613
·010	35·0	5·92	592
·011	39·5	6·28	571
·012	44·0	6·63	553
·013	50·0	7·07	544
·014	54·0	7·35	525
·015	59·0	7·68	512
·016	63·5	7·97	498
·017	69·5	8·34	490
·018	74·5	8·63	479

The difference between the numbers shown in these two Tables and in Table II. above, are probably due in part to true differences in the resistance of the air to electrical disruption; but variations in the electrometer, which was by no means of perfect construction,

may have sensibly influenced the results, especially as regards the differences between those shown in Table II. and those shown in Tables IV. and V., which agreeing on the whole closely with one another fall considerably short of the former.

TABLE VI. Summary of results reduced to absolute measure.

Col. 1. Length of spark in inches. s.	Col. 2. Electrostatic forces according to simple determinations of Dec. 13, 1859. $\frac{\sqrt{w}}{s} \times \frac{1}{5} \times \sqrt{\frac{32.2 \times 8\pi^*}{.187}}$ = X.	Col. 3. Electrostatic forces according to estimated average of various determinations. X.	Col. 4. Differences.	Col. 5. Pressures of electricity from either metallic surface balanced by air immediately before disruption, in grains weight per square foot †. $\frac{X^2}{8\pi \times 32.2}$
.001	...	11480	...	162800
.002	...	8000	...	79080
.003	...	6840	...	57810
.004	...	5820	...	41820
.005	...	5090	...	32010
.006	...	4700	...	27300
.007	4600	4530	+ .0070	26200
.008	...	4200	...	21830
.009	...	3990	...	19690
.010	...	3860	...	18390
.0105	3760	3770	-.0010	17450
.011	...	3720	...	17130
.0115	3620	3630	-.0010	16200
.012	...	3550	...	15580
.013	...	3480	...	14970
.014	3390	3390	.0000	14200
.015	...	3320	...	13580
.016	...	3260	...	13110
.017	3000	3140	-.0040	11800
.018	3190	3170	+.0020	12500
.024	3100	3000	...	11100
.0295	2820	2820	...	9830
.034	2740	2740	...	9250
.0385	2650	2650	...	8650
.041	2690	2690	...	8900
.0445	2650	2650	...	8640
.048	2600	2600	...	8350
.052	2530	2530	...	7910
.055	2510	2510	...	7770
.058	2490	2490	...	7630
.060	2500	2500	...	7720

* Distance between discs of absolute electrometer = $\frac{1}{60}$ foot = $\frac{1}{5}$ inch.

Area of each = .187 square foot.

Force of gravity at Glasgow on unit mass = 32.2 dynamical units of force; that is to say, generates in one second a velocity of 32.2 feet per second.

† This is most directly obtained by finding the force between the discs of the absolute electrometer per square foot, and reducing, according to the inverse proportion of squares of distances, to what it would have been if the distance between them had been equal to the length of the spark.

APPENDIX.

In order that the different expressions, "potential," "electromotive force," "electrostatic force," "pressure of electricity from a metallic surface balanced by air," used in the preceding statement, may be perfectly understood, I add the following explanations and definitions belonging to the ordinary elements of the mathematical theory of electricity.

1. *Measurement of quantities of electricity.*—The unit quantity of electricity is such a quantity, that, if collected in a point, it will repel an equal quantity collected in a point at a unit distance with a force equal to unity.

[In British absolute measurements the unit distance is one foot; and the unit force is that force which, acting on a grain of matter during a second of time, generates a velocity of one foot per second. The weight of a grain at Glasgow is 32·2 of these British units of force. The weight of a grain in any part of the earth's surface may be estimated with about as much accuracy as it can be without a special experiment to determine it for the particular locality, by the following expression:—

In latitude λ average weight of a grain

$$= 32\cdot088 \times (1 + \cdot005133 \times \sin^2 \lambda) \text{ British absolute units.}]$$

2. *Electric density.*—This term was introduced by Coulomb to designate the quantity of electricity per unit of area in any part of the surface of a conductor. He showed how to measure it, though not in absolute measure, by his proof plane.

3. *Resultant electric force at any point in an insulating fluid.*—The resultant force at any point in air or other insulating fluid in the neighbourhood of an electrified body, is the force which a unit of electricity concentrated at that point would experience if it exercised no influence on the electric distributions in its neighbourhood.

4. *Relation between electric density on the surface of a conductor, and electric force at points in the air close to it.*—According to a proposition of Coulomb's, requiring, however, correction, and first correctly given by Laplace, the resultant force at any point in the air close to the surface of a conductor is perpendicular to the surface and equal to $4\pi\rho$, if ρ denotes the electric density of the surface in the neighbourhood.

5. *Electric pressure from the surface of a conductor balanced by air.*—A thin metallic shell or liquid film, as for instance a soap-bubble,

if electrified, experiences a real mechanical force in a direction perpendicular to the surface outwards, equal in amount per unit of area to $2\pi\rho^2$, ρ denoting, as before, the electric density at the part of the surface considered. This force may be called either a repulsion (as according to the views of the eighteenth century school) or an attraction effected by tension of air between the surface of the conductor and the conducting boundary of the air in which it is insulated, as it would probably be considered to be by Faraday; but whatever may be the explanation of the *modus operandi* by which it is produced, it is a real mechanical force, and may be reckoned as in Col. 5 of the preceding Table, in grains weight per square inch or per square foot. In the case of the soap-bubble, for instance, its effect will be to cause a slight enlargement of the bubble on electrification with either vitreous or resinous electricity, and a corresponding collapse on being perfectly discharged. In every case we may regard it as constituting a deduction from the amount of air-pressure which the body experiences when unelectrified. The amount of this deduction being different in different parts according to the square of the electric density, its resultant action on the whole body disturbs its equilibrium, and constitutes in fact the resultant electric force experienced by the body.

6. *Collected formulæ of relation between electric density on the surface of a conductor, electric diminution of air-pressure upon it, and resultant force in the air close to the surface.*—Let, as before, ρ denote the first of these three elements, let p denote the second reckoned in units of force per unit of area, and let R denote the third. Then we have

$$R=4\pi\rho,$$

$$p=2\pi\rho^2=\frac{1}{8\pi}R^2.$$

7. *Electric potential.*—The amount of work required to move a unit of electricity against electric repulsion from any one position to any other position, is equal to the excess of the electric potential of the first position above the electric potential of the second position.

Cor. 1. The electric potential at all points close to the surface of an electrified metallic body has one value, since an electrified point, possessing so small a quantity of electricity as not sensibly to influence the electrification of the metallic surface, would, if held near the surface in any locality, experience a force perpendicular to the surface in its neighbourhood.

Cor. 2. The electric potential throughout the interior of a hollow metallic body, electrified in any way by external influence, or, if insulated, electrified either by influence or by communication of electricity to it, is constant, since there is no electric force in the interior in such circumstances.

[It is easily shown by mathematical investigation, that the electric force experienced by an electric point containing an infinitely small quantity of electricity, when placed anywhere in the neighbourhood of a hollow electrified metallic shell, gradually diminishes to nothing if the electric point be moved gradually from the exterior through a small aperture in the shell into the interior. Hence the one value of the potential close to the surface outside, mentioned in *Cor. 1*, is equal to the constant value throughout the interior mentioned in *Cor. 2*.]

8. *Interpretation of measurement by electrometer.*—Every kind of electrometer consists of a cage or case containing a moveable and a fixed conductor, of which one at least is insulated and put in metallic communication, by what I shall call the principal electrode passing through an aperture in the case or cage, with the conductor whose electricity is to be tested. In every properly constructed electrometer, the electric force experienced by the moveable part in a given position cannot be electrically influenced except by changing the difference of potentials between the principal electrode and the uninsulated conductor or conducting system in the electrometer. Even the best of ordinary electrometers hitherto constructed do not fulfil this condition, as the inner surface of the glass of which the whole or part of the enclosing case is generally made, is liable to become electrified, and inevitably does become so when any very high electrification is designedly or accidentally introduced, even for a very short time; the consequence of which is that the moving body will generally not return to its zero position when the principal electrode is perfectly disinsulated. Faraday long ago showed how to obviate this radical defect by coating the interior of the glass case with a fine network of tinfoil; and it seems strange that even at the present day electrometers for scientific research, as for instance for the investigation of atmospheric electricity, should be constructed with so bad and obvious a defect uncured by so simple and perfect a remedy. When it is desired to leave the interior of the electrometer as much light

as possible, and to allow it to be clearly seen from any external position with as little embarrassment as possible, a cage made like a bird's cage, with an extremely fine wire on a metal frame, inside the glass shade used to protect the instrument from currents of air, &c., may be substituted with advantage for the tinfoil network lining of the glass. It appears therefore that a properly constructed electrometer is an instrument for measuring, by means of the motions of a moveable conductor, the difference of potentials of two conducting systems insulated from one another, of one of which the case or cage of the apparatus forms part. It may be remarked in passing, that it is sometimes convenient in special researches to insulate the case or cage of the apparatus, and allow it to acquire a potential differing from that of the earth, and that then, as always, the subject of measurement is the difference of potentials between the principal electrode and the case or cage, while in the ordinary use of the instrument the potential of the latter is the same as that of the earth. Hence we may regard the electrometer merely as an instrument for measuring differences of potential between two conducting systems mutually insulated; and the object to be aimed at in perfecting any kind of electrometer (more or less sensitive as it may be, according to the subjects of investigation for which it is to be used), is, *that accurate evaluations in absolute measure, of differences of potential, may be immediately derivable from its indications.*

9. *Relation between electrostatic force and variation of electric potential.*—§ 7, otherwise stated, is equivalent to this:—The average component electrostatic force in the straight line of air between two points in the neighbourhood of an electrified body is equal to their difference of potentials divided by their distance. In other words, the rate of variation of electric potential per unit of length in any direction, is equal to the component of the electrostatic force in that direction. Since the average electrostatic force in the line joining two points at which the values of the potential are equal, is nothing, the direction of the resultant electrostatic force at any point must be perpendicular to the equipotential surface passing through that point; or the lines of force (which are generally curves) cut the series of equipotential surfaces at right angles. The rate of variation of potential per unit of length along a line of force is therefore equal to the electrostatic force at any point.

10. *Stratum of air between two parallel or nearly parallel plane or curved metallic surfaces maintained at different potentials.*—Let a denote the distance between the metallic surfaces on each side of the stratum of air at any part, and V the difference of potentials. It is easily shown that the resultant electrostatic force is sensibly constant through the whole distance, from the one surface to the other; and being in a direction sensibly perpendicular to each, it must (§ 9) be equal to $\frac{V}{a}$. Hence (§ 4) the electric density on each of the opposed surfaces is equal to $\frac{V}{4\pi a}$. This is Green's theory of the Leyden phial.

11. *Absolute Electrometer.*—As a particular case of No. 10, let the discs be plane and parallel; and let the distance between them be small in comparison with their diameters, or with the distance of any part of either from any conductor differing from it in potential. The electric density will be uniform over the whole of each of the opposed surfaces and equal to $\frac{V}{4\pi a}$, being positive on one and negative on the other; and in all other parts of the surface of each the electrification will be comparatively insensible. Hence the force of attraction between them per unit of area (§§ 5 and 6) will be $\frac{V^2}{8\pi a^2}$, if A denote the area of either of the opposed surfaces; the whole force of attraction between them is therefore $A\frac{V^2}{8\pi a^2}$. Hence, if the observed force be equal to the weight of w grains at Glasgow, we have

$$32\cdot2 \times w = A \frac{V^2}{8\pi a^2},$$

and therefore

$$V = a \sqrt{\frac{32\cdot2 \times 8\pi \times w}{A}}.$$

Addition, dated April 12, 1860.

Experiments on precisely the same plan as those of Table I. December 13, have been repeated by the same two experimenters, with different distances from $\cdot 3$ to $\cdot 6$ of an inch between the plates of the absolute electrometer, and results have been obtained confirming the general character of those shown in the preceding Tables.

The absolute evaluations derived from these later series, must be more accurate than those deduced above from the single series of December 13, when the distance between the plates in the absolute electrometer was only $\cdot 2$ of an inch. I therefore by permission add the following Table of absolute determinations :—

Length of spark in inches. s.	Electrostatic forces according to estimated average of determina- tions of February 15, 23, 28, and 29, and March 2. X.
$\cdot 0034$	5793
$\cdot 005$	5574
$\cdot 006$	5688
$\cdot 0075$	4862
$\cdot 0111$	4351
$\cdot 0161$	3287
$\cdot 0222$	3125
$\cdot 023$	3029
$\cdot 0271$	3055
$\cdot 0356$	2925
$\cdot 0416$	2865
$\cdot 0522$	2841

These results, as well as those shown in the preceding Tables, demonstrate a much less rapid variation with distance, of the electrostatic force preceding a spark, at the greater than at the smaller distances. It seems most probable that at still greater distances the electrostatic force will be found to be sensibly constant, as it was certainly expected to be at all distances. The limiting value to which the results shown in the last Table seem to point must be something not much less than 2800. This corresponds to a pressure of 9600 grains weight per square foot. We may therefore conclude that the ordinary atmospheric pressure of 14,798,000 grains per square foot, is electrically relieved by the subtraction of 9600 on two very slightly convex metallic surfaces, at a distance of $\frac{1}{20}$ th of an inch or more, before the air between them is cracked and a spark passes. By taking into account the result of my preceding communication to the Royal Society, we may also conclude that a Daniell's battery of 5510 elements can produce a spark between two slightly convex metallic surfaces at $\frac{1}{20}$ th of an inch asunder in ordinary atmospheric air.